Proposal for an experiment to search for Randall-Sundrum type corrections to Newton's law of gravitation

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String theory, as well as the string inspired brane-world models such as the Randall-Sundrum (RS) one, suggest a modification of Newton's law of gravitation at small distance scales. Search for modifications of standard gravity is an active field of research in this context. It is well known that short range corrections to gravity would violate the Newton-Birkhoff theorem. Based on calculations of RS type non-Newtonian forces for finite size spherical bodies, we propose a torsion balance based experiment to search for the effects of violation of this theorem valid in Newtonian gravity as well as the general theory of relativity. We explain the main principle behind the experiment and provide detailed calculations suggesting optimum values of the parameters of the experiment. The projected sensitivity is sufficient to probe the Randall-Sundrum parameter up to 10 microns.

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Einstein's theory of gravitation is the theory of space-time where the gravitational field is associated with the space-time metric and curvature[1]. Although phenomenologically an extremely successful theory, attempts to quantize this geometric field have so far led to no decisive progress. This difficulty has led many investigators to consider higher dimensional theories in the hope that such attempts may help to ultimately arrive at the quantum theory of gravitation in $(3+1)$ dimensions. String theory [2] and string inspired higher dimensional theories such as the brane-world models [3] are examples of such attempts. These theories suggest that the higher dimensional effects would generally show up as a short range correction to Newton's law of gravitation [2, 3]. Direct astronomical observations and laboratory experiments had ruled out corrections with range larger than a few millimeters even before the recent surge of interest in higher dimensional theories. This leaves possibility of corrections to Newton's law of gravity at millimeter and submillimeter length scales [5]. Recent experiments are steadily progressing to probe length scales down to 10 microns.

In this paper, we will be concerned only with the 5-dimensional Randall-Sundrum (RS) brane-world model because it is simple and elegant, and it brings out the feature of the correction to Newtonian gravity in a transparent manner [3, 4]. The RS corrected potential is given by

$$U(r) = -\frac{Gm}{r}\left(1 + \frac{l_s^2}{r^2}\right) \tag{1}$$

where the Randall-Sundrum parameter $l_s^2 = \frac{2}{3}l^2$, l is the curvature scale of 5-dimensional anti-deSitter space-time, G is Newton's constant of gravity, m is mass and r is the distance in 3-space. It turns out that these corrections do not have any astrophysical significance(see Ref.[6] and references there in). This leads us to conclude that, other than accelerator based high energy experiments, direct observation of this force in laboratories is the only way to test the presence of such correction terms. We propose here a torsion balance based experiment.

In the last two decades, several laboratory based experiments have been carried out to verify the presence of corrections to Newtonian gravity. The results in these experiments are generally parameterized with an additional Yukawa interaction [5],

$$V(r) = -\frac{Gm_1m_2}{r}\Big[1 + \alpha \;\; exp(-\frac{r}{\lambda})\Big] \tag{2}$$

α being the strength of the additional interaction relative to Newtonian gravity and λ the range of the interaction. These experiments set limit on the strength α for distance scale λ, implying the absence of additional force whose strength relative to Newtonian gravity, at distance scale λ, is equal to or larger than α. Even before the provocations from string inspired models, in the years when a hypothetical 'fifth force' was searched for, experimentalists had put stringent constraints in the $\alpha - \lambda$ plane at length scale down to a few mm [7, 8, 9]. University of California at Irvine group used "null-geometry" for torsion balance experiment and set limit in the range : $\alpha = 10^{-2}$ at $\lambda = 3mm$ to $\alpha = 10^{-4}$ at $\lambda = 3cm$ [7]. More recently, the Washington University group operated a specially designed "missing mass" torsional pendulum experiment and set limit in the range: $\alpha = 10$ at $\lambda = 100$ microns to $\alpha = 10^{-2}$ at $\lambda = 3mm$ [10]. "Cantilever" and "micro-cantilever" based experiments have been carried out by the Colorado group and the Stanford group respectively with constraints below 100 microns[11]. There are also some experiments based on the measurement of Casimir effect [12](see Refs.[9, 13] for details).

The experiment we propose shares some features of the "null-geometry" experiment of the University of California, Irvine group. But we stress the importance of bulk

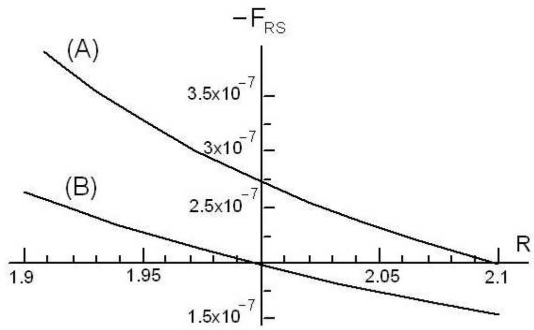

FIG. 1: Force between two spherical balls only due to the RS corrections as a function of distance between their centres of masses. (A) Force between 100 gm of silver ball and 10 gm of gold ball. (B) Force between 100 gm of gold ball and 10 gm of gold ball. Force is given in dyne and distance in cm . The RS parameter $l_s = 1mm$ in both cases.

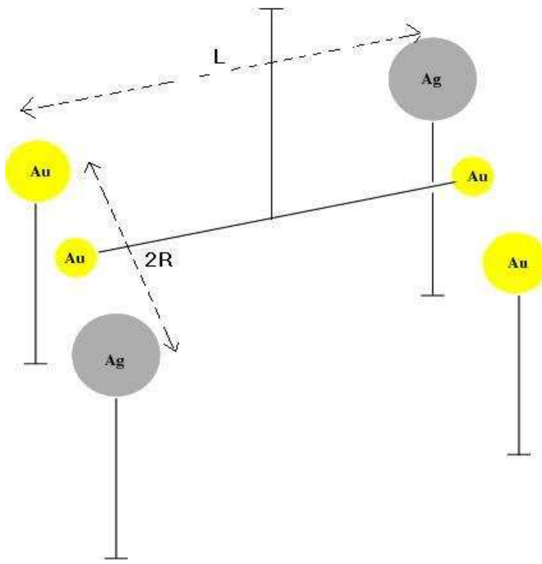

FIG. 2: The experimental set up to measure the shift in the equilibrium position of the torsion balance. The ball at one end of the balance is subjected to the combined force of Newtonian gravity and the RS correction terms of the two balls symmetrically fixed at the same end of the balance. Force on this ball due to the other two balls fixed at the opposite end of the balance is negligible. $L = 20cm$. R varies with l_s. For $l_s = 1\ mm$, $R = 2\ cm$.

spherical body in the case of Randall-Sundrum gravity. The main idea is that the Randall-Sundrum potential, like any other short range potential, violates Newton-Birkhoff theorem. This theorem, in our context, means that the effects of Newtonian gravity as well as of general relativity of a spherically symmetric body depend only on the mass and is independent of its size and the density of the material [1]. With R-S potential, however, a spherically symmetric body does not behave as a point source of gravity and the potential as well force depends on density and size. Our proposed experiment is intended to search for the quantitative and qualitative outcome of violation of this theorem in the case of this single parameter model. We show that the short range corrections can lead to a measurable effect for the numerical value of RS parameter l_s at least up to 10 microns.

In the following, we derive, in details, the Randall-Sundrum (RS) interaction potential between two solid spheres of finite but different radii and densities, separated by a distance, R, between their centers. The RS potential $\phi_{RS}(r)$ of a spherical ball of radius a and constant density ρ, at a distance $r > a$ is

$$\begin{aligned}\phi_{RS}(r) &= -Gl_s^2 \int \frac{\rho(\vec{r'})d^3\vec{r'}}{|\vec{r}-\vec{r'}|^3} \\ &= -2\pi G\ l_s^2\ \rho\ \left[\ ln\frac{r+a}{r-a}\ -\ \frac{2a}{r}\right] \qquad (3)\end{aligned}$$

which shows that the short range RS correction to gravity violates Newton-Birkhoff theorem.
The force on a point mass m, is given by,

$$\begin{aligned}f_{RS} &= -m\nabla\Phi_{RS}(r) = -3mGl_s^2 \times \left[\frac{M}{r^2(r^2-a^2)}\right] \\ &= -2\pi mGl_s^2\ \times \frac{\rho}{\epsilon} \qquad (4)\end{aligned}$$

where the distance of the point mass $r = a+\epsilon$. Close to the surface of the ball, the force is large and depends only on the density of the source material. But away from the surface it falls off very fast. Let us now consider two spherical balls of equal mass M but different radii a_1 and a_2, and densities ρ_1 and ρ_2, $\rho_1 > \rho_2$, $a_2 > a_1$. Distance between the centers of the two spheres is $2r$. A point mass m is placed at the midpoint on the line joining their centers. The Newtonian force of spheres on the point mass test particles balance each other. If f_1 and f_2 are forces due to short range RS interaction, then

$$\frac{f_2}{f_1} = \frac{r^2-a_1^2}{r^2-a_2^2} > 1 \qquad (5)$$

In the real experimental situations both the source mass and the test mass have finite sizes. In what follows we shall calculate the RS interaction potential between two spheres with radii a, b and densities ρ_a, ρ_b respectively. Let the distance between the centers of the spheres be R. Using Eq.(3), the RS correction to the potential due to the two balls can be computed as

$$\Phi_{RS}(R) = -2\pi\ \rho_a\ \rho_b\ G\ l_s^2 \times \Bigg\{ \int_0^b\ r^2\ dr \int_0^\pi\ sin\theta d\theta$$
$$\int_0^{2\pi} d\phi\Bigg(ln\frac{|\vec{R}-\vec{r}|+a}{|\vec{R}-\vec{r}|-a} - \frac{2a}{|\vec{R}-\vec{r}|}\Bigg)\Bigg\}$$

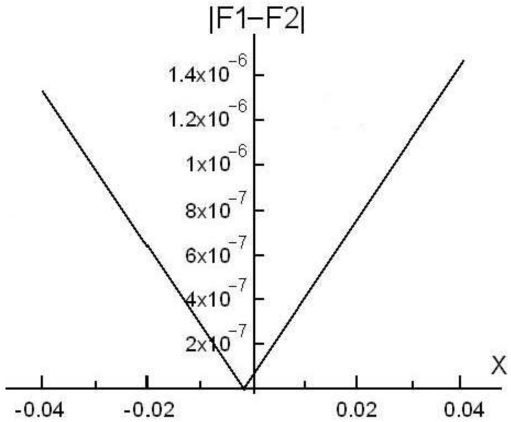

FIG. 3: Combined force of Newtonian gravity and the RS-correction terms: The vertical axis is the absolute value of the difference of combined forces (in dynes) due to the fixed source masses, the 100 gm gold ball and the 100 gm silver ball on the 10 gm gold ball of the torsion balance. The horizontal axis, X, is the distance (in cms) of the centre of mass of the 10 gm gold ball from geometric midpoint between the centres of the source masses. The RS parameter $l_s = 1mm$. Details given in the text.

Integration over angles ϕ, θ, and radial parameter r gives

$$\Phi_{RS}(R) = -\frac{2\pi^2\rho_a\rho_b G l_s^2}{R}\Bigg(\Bigg\{\Bigg[\frac{1}{4}(a^4+b^4) - \frac{1}{2}a^2b^2 + \frac{1}{2}R^2(a^2+b^2) - \frac{R^4}{12}\Bigg]\ ln\frac{R^2-(a+b)^2}{R^2-(a-b)^2}\Bigg\} + \Bigg\{\frac{2R}{3}\Bigg[a^3\ ln\frac{(R+b)^2-a^2}{(R-b)^2-a^2} + b^3\ ln\frac{(R+a)^2-b^2}{(R-a)^2-b^2}\Bigg]\Bigg\} - \Bigg\{a^3b + \frac{1}{3}R^2ab + ab^3\Bigg\}\Bigg) \quad \dots \dots \dots \tag{6}$$

The point mass test particle limit is obtained when $b << a$ and $b << R-a$, $M_b = \frac{4\pi}{3}b^3\rho_b$, In this limit, the expression above takes the form,

$$\Phi_{RS}(R,a,b) = -2\pi G l_s^2 \rho_a M_b\Bigg(ln\frac{R+a}{R-a} - \frac{2a}{R}\Big[1 + \mathcal{O}\Big(\frac{b^2}{R^2}(1-a/R)^{-4}\Big)\Big]\Bigg) \tag{7}$$

The potential given by Eq.(6) and the force generated by it monotonically decrease to a finite value in the limit $R \rightarrow a+b$. Thus the force as well the potential are finite when the balls touch each other. In Fig.1 , we provide plots of the forces due to the correction term. The vertical axis in the figure is $-F_{RS}$, the absolute value of the force (in dynes), while the horizontal axis is the distance between the centers of masses of the balls (in cm). Plot (A) is the force, $-F_{RS}$, between 100 gm of silver ball and 10 gm of gold ball, while plot (B) is the force, $-F_{RS}$, between 100 gm gold ball and 10 gm gold ball. There is some difference between the forces in the two cases considered. In addition, the forces do not fall off too fast with the increase of distance between the balls within the range favorable for a torsion balance experiment. These are the features of RS corrections that we want to exploit for our experiment. We emphasize that the magnitudes of the forces and the relevant size scales are suitable for a torsion balance experiment. An increase in density contrast of the source materials or contrast in mass/density of the sources and of the test body does not lead to any additional advantage.

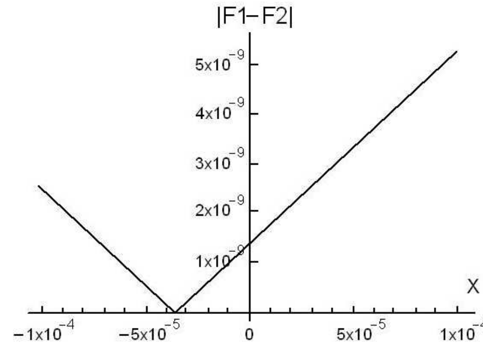

FIG. 4: The plot is the same as the plot in Fig.3 with RS parameter $l_s = 100\ microns$. Details given in the text.

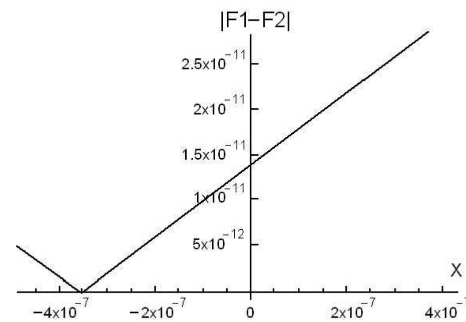

FIG. 5: This plot is the same as the plot in Fig.3 with RS parameter $l_s = 10\ microns$. Details given in the text.

A sketch of the scheme of the experiment is given in Fig.2. We have four balls of 100 gm each placed in a planar rectangular configuration in such a way that the centers of mass of the balls are on the horizontal plane. The silver balls are diagonally opposite to each other and so are the gold balls (radii of gold and silver balls are 1.073 cm and 1.315 cm respectively). Along the longer axis of the rectangle the distance, L, between the centers of mass of the silver and gold balls is 20 cm, and along the shorter axis the distance, $2R$, is 4 cm, a torsion balance hangs in the middle, parallel to the longer

axis of the rectangle. At each end of the hanging bar of the torsion balance are attached two gold balls of 10 gm each with radius 0.498 cm. The distance between the centers of mass of these balls is 20 cm. The torsion coefficient of the suspension wire can be taken to be about 0.1 $dyne\ cm/radian$. In the absence of RS-correction term, the Newtonian force of the 100 gm silver and gold balls create unstable equilibrium point in middle of the shorter axis of the rectangle at a distance of $R = 2\ cm$ from either of the balls. In the presence of RS-correction, the effect mentioned in the earlier paragraph would come into play and the combined effect of Newtonian gravity and the RS-corrections would shift the location of the unstable equilibrium point. Then the torsion balance would oscillate about this shifted minimum of its harmonic potential.

In Fig.3, we plot the absolute value of the difference of forces (in dynes) due to the combined effect of Newtonian gravity and the RS-correction terms of the fixed source masses, the 100 gm gold ball and the 100 gm silver ball, on the 10 gm gold ball of the torsion balance as a function of its distance (in cm) from the geometric midpoint which is situated at a distance of $R = 2\ cm$ from the centre of either of the source masses. It should be noted that the unstable equilibrium position, where the combined force is zero, moves by 20 $microns$ towards the 100 gm gold ball. In the experimental configuration under consideration, this shift of the equilibrium position towards the higher density ball is a qualitative effect. Therefore, some systematic experimental uncertainty can be eliminated by interchanging the positions of the 100 gm gold and the silver balls. The equilibrium position should again move towards the gold ball. The position of the unstable equilibrium is found by locating the changed equilibrium position of the torsion balance. The shift in the equilibrium position can be increased by decreasing the distance between the fixed 100 gm gold and the 100 gm silver balls along the shorter axis rectangular configuration but leaving the distance along the longer axis unchanged. For example, a distance of 3.8 cm with midpoint at 1.9 cm, the equilibrium position shifts by 35 microns. A further decrease can give a shift of about 50 microns, after which the atomic forces start to interfere.

The accuracy of the angular shift that can be measured with standard technology in a torsion balance experiment is below $10^{-9}\ \ rad/\sqrt{Hz}$ which for our configuration amounts to a distance shift of the end point of the balance of about $10^{-8}\ \ cm$ which is several order of magnitude smaller than the shift in the case when RS parameter $l_s = 0.1\ cm$. Systematic effects due to Newtonian gravity gradients arising from errors of about 5 microns in the position of the source masses, and due to the deviations from their sphericity and density homogeneity at the level of 10^{-3} generate less than 100 nm shift in the equilibrium position of the test mass [14]. The small drift of the torsion balance, amounting to 1 $microradian$ per hour, can also be corrected at this level in repeated measurements [15]. Therefore, achieving required sensitivity to detect RS corrections for $l_s = 100\ microns$ is not difficult. To probe RS corrections for $l_s = 10\ microns$ the masses have to be located accurate to less than 1 $micron$ and the drift should be corrected at 1% level, which is feasible but requires considerable care in experimental design. This sets the lower limit on the value of RS parameter that can be probed with some reasonable degree of confidence to about $l_s = 10\ microns$. This can be inferred from figures Figs.4 & 5. These figures correspond to the case when the length of shorter rectangular axis in the setup in Fig.2 is 3.8 cm.

We have discussed in this paper the basic principle, feasibility and the schematic of the experiment. A torsion balance experiment along the lines discussed in this paper is under active consideration at the Tata Institute of Fundamental research, Mumbai.

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